Quantum reflection of massless neutrinos

from a torsion-induced potential

M. Alimohammadi $^{a-1}$, A. Shariati b,c

- ^a Department of Physics, University of Tehran, North Karegar, Tehran, Iran
- ^b Institute for Advanced Studies in Basic Physics , P.O.Box 159 , Gava Zang , Zanjan 45195 , Iran
 - ^c Institute for Studies in Theoretical Physics and Mathematics, P.O.Box 19395-5531, Tehran, Iran

Abstract

In the context of the Einstein–Cartan–Dirac model, where the torsion of the space–time couples to the axial currents of the fermions, we study the effects of this quantum–gravitational interaction on a massless neutrino beam crossing through a medium with high number density of fermions at rest. We calculate the reflection amplitude and show that a specific fraction of the incident neutrinos reflects from this potential if the polarization of the medium is different from zero. We also discuss the order of magnitude of the fermionic number density in which this phenomenon is observable, in other theoretical contexts, for example the strong–gravity regime and the effective field theory approach.

¹e-mail:alimohmd@theory.ipm.ac.ir

1 Introduction

Studying the physics of gravity with torsion and especially the interaction of torsion with a spinor field, has attracted attention for a long time [1-5]. Recently the interest in these theories has increased because of formal development of string theory. The low–energy limit of string theory has a an antisymmetric tensor field of the third rank which is usually associated with torsion of space—time [6].

One of the important examples of torsion–fermion interaction, is the interaction of torsion of space—time with neutrinos. These kinds of investigations go back also to several years ago. For example in ref. [5] (see also [7]), the effect of torsion on neutrino oscillation has been studied by assuming that the torsion eigenstates, i.e., the eigenstate of the interaction part of the Hamiltonian, are different from the weak interaction eigenstates. Recently, the contribution of the torsion of space—time on standard neutrino oscillation has been studied in the context of Einstein—Cartan—Dirac theory and for the case where the mass eigenstates are different from the weak interaction eigenstates (which is assumed to be the same as the torsion eigenstates). The situations in which the torsion effect on neutrino oscillation is as important as the neutrino mass effect have also been discussed [8].

In this paper, we want to study the effect of torsion-neutrino interaction potential on the propagation of a massless neutrino beam crossing through a region (object) which has torsion. As we will see, the axial-currents of all kinds of fermions, including the neutrinos itself, couple to the totally antisymmetric part of contorsion, and this coupling produce a potential barrier with width equal to the radius of that object, if the average spin component of the fermions, in some direction, is different from zero. We will show that this kind of potential, affects the incident massless neutrino flux and reflect a specific fraction of it. Note that this reflection is not trivial, because if one considers a constant potential barrier in the Dirac problem of a massless particle, it can be shown that this barrier can not affect the incident beam and therefore a constant potential barrier is transparent for massless particles. We think that this is an interesting phenomenon, as the only known interaction which can affect an incident massless neutrino beam is the standard weak interaction, and now we see that this completely gravitational effect can also produce a quantum mechanical reflection. On the other hand, this effect predicts the lack of neutrino flux when crossing through a region which has a high density polarized fermions.

The plan of the paper is as follows. In section 2, we briefly discuss the Einstein–Cartan–Dirac theory and derive the above mentioned interaction term, and in section 3, we solve the simple problem of crossing a massless neutrino beam through this gravitational potential barrier. At the end, we discuss the order of magnitude of this effect in different theoretical frameworks.

2 Einstein-Cartan-Dirac theory

Consider a four dimensional manifold U^4 which is specified by two independent tensor fields, the Riemannian metric $g_{\mu\nu}$ and the connection $\Gamma^{\mu}_{\alpha\beta}$ where its most general form, compatible with $g_{\mu\nu}$, is

$$\Gamma^{\alpha}_{\mu\nu} = \begin{Bmatrix} {\alpha} \\ {\mu\nu} \end{Bmatrix} + K^{\alpha}_{\mu\nu},\tag{1}$$

where

$$\begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} = \frac{1}{2} g^{\alpha\beta} \left(-g_{\mu\nu,\beta} + g_{\beta\mu,\nu} + g_{\nu\beta,\mu} \right), \tag{2}$$

is the usual Christoffel symbol, and $K^{\alpha}_{\mu\nu}$ is the contorsion of U^4 . The contorsion tensor is related to the torsion tensor as follows

$$K^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta} \left(T_{\beta\mu\nu} + T_{\mu\beta\nu} + T_{\nu\beta\mu} \right), \tag{3}$$

and the torsion itself is the antisymmetric part of the connection

$$T^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}.\tag{4}$$

In this way, it is obvious that the differential geometry of U^4 is determined by two independent tensors $g_{\mu\nu}$ and $K^{\alpha}_{\mu\nu}$ (or equivalently $T^{\alpha}_{\mu\nu}$).

If one decomposes the contorsion $K_{\alpha\mu\nu}$ as following

$$K_{\alpha\mu\nu} = \frac{1}{3} \left(g_{\alpha\mu} \tau_{\nu} - g_{\nu\mu} \tau_{\alpha} \right) + \frac{1}{2} \mathcal{A}^{\sigma} \varepsilon_{\sigma\alpha\mu\nu} + U_{\alpha\mu\nu}, \tag{5}$$

it can be shown that the independent vectors τ_{μ} and A^{σ} satisfy

$$\tau_{\mu} = g^{\alpha\beta} K_{\alpha\beta\mu},\tag{6}$$

$$\mathcal{A}^{\sigma} = \frac{1}{3} \varepsilon^{\sigma \alpha \mu \nu} K_{\alpha \mu \nu},\tag{7}$$

and $U_{\alpha\mu\nu}$ has the following properties

$$U_{\alpha\mu\nu} = -U_{\nu\mu\alpha}, \quad g^{\alpha\mu}U_{\alpha\mu\nu} = 0, \quad \varepsilon^{\sigma\alpha\mu\nu}U_{\alpha\mu\nu} = 0. \tag{8}$$

In the above equation, $\varepsilon_{\sigma\alpha\mu\nu}$ is the totally antisymmetric pseudo-tensor of rank 4. Now if we calculate the scalar curvature of U^4 , using (1) and (5-8), obtain

$$R = {}^{0}R - \frac{2}{\sqrt{g}}\partial_{\kappa}(\sqrt{g}\tau^{\kappa}) - \left(\frac{2}{3}\tau^{2} + \frac{3}{2}\mathcal{A}^{2} + U_{\alpha\mu\nu}U^{\mu\nu\alpha}\right),\tag{9}$$

where 0R is the scalar curvature of the same manifold but with vanishing torsion, i.e., $\Gamma^{(0)}{}^{\alpha}_{\mu\nu} = \left\{^{\alpha}_{\mu\nu}\right\}$, and $\sqrt{g} = [-\det(g_{\mu\nu})]^{1/2}$.

If we couple a spin-1/2 particle to this space-time, the resulting theory is known as the Einstein-Cartan-Dirac (ECD) theory, and its action is defined by

$$I_{\text{ECD}} = -\frac{c^3}{16\pi G} \int d^4x \sqrt{g} R + \sum_j \int d^4x \sqrt{g} (-\hbar) \bar{\psi}_j \left(e_a^\mu \gamma^a (\partial_\mu + \Gamma_\mu) + i \frac{m_j c}{\hbar} \right) \psi_j, \tag{10}$$

where a is tetrad index and Γ_{μ} is the spin connection

$$\Gamma_{\mu} = -\frac{i}{8} [\gamma^a, \gamma^b] e^{\nu}_a e_{b\nu;\mu},\tag{11}$$

and the sum is over different types of fermions. In the above equation, ; μ denotes the covariant derivative on U^4

$$e_{b\nu;\mu} := e_{b\nu,\mu} - \Gamma^{\lambda}_{\nu\mu} e_{b\lambda} = e_{b\nu,\mu} - \begin{Bmatrix} \lambda \\ \nu\mu \end{Bmatrix} e_{b\lambda} - K^{\lambda}_{\nu\mu} e_{b\lambda}. \tag{12}$$

It can be easily shown that the variation of the action (10) with respect to $\bar{\psi}_j$, A^{μ} , τ^{μ} , and $U_{\alpha\mu\nu}$ leads to the following equations of motion, respectively, [8,9]

$$\gamma^{\mu}\partial_{\mu}\psi_{j} + i\frac{m_{j}c}{\hbar}\psi_{j} + \frac{i}{4}\gamma_{5}A_{\mu}\gamma^{\mu}\psi_{j} = 0, \tag{13}$$

$$A^{\mu} = \frac{12\pi\hbar G}{c^3} \sum_{j} (J_j)_5^{\mu},\tag{14}$$

$$\tau^{\mu} = 0, \tag{15}$$

$$U_{\alpha\mu\nu} = 0, \tag{16}$$

where $(J_j)_5^{\mu} = \bar{\psi}_j \gamma^{\mu} \gamma_5 \psi_j$ is the axial–current of the j-th Dirac field.

Therefore, in the context of ECD theory:

- 1) the axial–currents of the fermions of a region are the source of the torsion of the space–time of that region,
- 2) when a beam of neutrinos crossing through this region, interacts with the fermionic matter via $\mathcal{L}_{int} = \frac{i}{4} A_{\mu} \bar{\psi}_{neutr.} \gamma_5 \gamma^{\mu} \psi_{neutr.}$, even the neutrinos are massless.

3 Massless neutrino reflection

In order to study the effect of \mathcal{L}_{int} on a neutrino beam, let us consider the simple case where the massless neutrino beam crosses through a spherical object with radius R and with fermionic matter all in at rest. So we must first calculate the vector A^{μ} of this medium. In chiral representation, the spinor wavefunction of a particle of mass m and zero momentum, $\mathbf{p} = 0$, is $\psi = \sqrt{\rho} \begin{pmatrix} \chi \\ \chi \end{pmatrix} e^{-iEt/\hbar}$, where ρ is the number density of the particles and χ is a two-component spinor that must be chosen. We assume that the spin of the particles

is aligned along the unit vector $\hat{\mathbf{s}}$ which is characterized by polar and azimuthal angles β and α , respectively. We choose χ to be the eigenspinor of $\sigma.\hat{\mathbf{s}}$, i.e., $\sigma.\hat{\mathbf{s}}\chi = \chi$, so $\chi = \left(\frac{\cos(\beta/2)e^{-i\alpha/2}}{\sin(\beta/2)e^{i\alpha/2}}\right)$. In this way $J_5^{\mu} = \bar{\psi}\gamma^{\mu}\gamma_5\psi$ becomes $(0, -2\rho\hat{\mathbf{s}})$, and therefore

$$A^{\mu} = \left(0, -\frac{24\pi\hbar\rho G}{c^3}\hat{\mathbf{s}}\right). \tag{17}$$

If there are more than one kind of fermion in the medium, we must also add their contributions to the above relation. In deriving (17), we assume that all the fermions are polarized along the same direction $\hat{\mathbf{s}}$. If the situation is not so, we must put the average value of spins in (17). Therefore \mathcal{L}_{int} vanishes for completely random distribution of spins, i.e., $\mathcal{L}_{int} = 0$ for unpolarized medium.

Now working in chiral representation, the equation of motion (13) leads to the following Hamiltonian equation for the wavefunction of massless $(m_{\nu} = 0)$ neutrinos

$$(H_0 + V_T)u(\mathbf{p}) = Eu(\mathbf{p}),\tag{18}$$

where $u(\mathbf{p})$ is defined in $\psi(\mathbf{r},t) = e^{-i/\hbar(Et-\mathbf{p}.\mathbf{r})}u(\mathbf{p})$. Also

$$H_0 = \begin{pmatrix} -c\sigma \cdot \mathbf{p} & 0\\ 0 & c\sigma \cdot \mathbf{p} \end{pmatrix},\tag{19}$$

and

$$V_T = -\frac{\hbar c}{4} \begin{pmatrix} \sigma.\mathbf{A} & 0\\ 0 & \sigma.\mathbf{A} \end{pmatrix} = K \begin{pmatrix} \sigma.\hat{\mathbf{s}} & 0\\ 0 & \sigma.\hat{\mathbf{s}} \end{pmatrix}, \tag{20}$$

where we have used the expression (17) for A^{μ} , and K is the coupling constant of this model (ECD model) and is equal to

$$K_{\rm ECD} = \frac{6\pi\rho G\hbar^2}{c^2}.$$
 (21)

Note that in eq.(20), σ is the spin vector of the incoming neutrinos, while $\hat{\mathbf{s}}$ is the polarization direction of the target fermions. For simplicity, we take the momentum direction of neutrinos as z-direction and the polarization direction of target particles as x-direction. It can be shown that the z-component of $\hat{\mathbf{s}}$ does not contribute in neutrino scattering, so if the neutrinos and the target fermions have the same polarization direction, the fermionic medium becomes transparent for the incident massless neutrino beam.

In this way we find the following solution for different regions: for $z \leq 0$ region

$$\psi_{1} = e^{ipz} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + Ae^{-ipz} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + Be^{-ipz} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{22}$$

for $0 \le z \le R$ region

$$\psi_2 = Ce^{iqz} \begin{pmatrix} \alpha \\ 1 \\ 0 \\ 0 \end{pmatrix} + De^{iqz} \begin{pmatrix} 0 \\ 0 \\ 1 \\ \alpha \end{pmatrix} + Ee^{-iqz} \begin{pmatrix} 1 \\ \beta \\ 0 \\ 0 \end{pmatrix} + Fe^{-iqz} \begin{pmatrix} 0 \\ 0 \\ \beta \\ 1 \end{pmatrix}, \tag{23}$$

and for $z \geq R$ region

$$\psi_3 = Ge^{ipz} \begin{pmatrix} 0\\1\\0\\0 \end{pmatrix} + He^{ipz} \begin{pmatrix} 0\\0\\1\\0 \end{pmatrix}. \tag{24}$$

In the above equations A, \dots, H are coefficients that must be determined, E = pc is the energy of the incident neutrino, q is defined in

$$E = \sqrt{K^2 + c^2 q^2},\tag{25}$$

and α and β are

$$\alpha = \frac{E - cq}{K},$$

$$\beta = \frac{K}{E + cq}.$$
(26)

Writing down the continuities of the solutions in z = 0 and z = R, determine the coefficients as following

$$B = D = F = H = 0, (27)$$

so the neutrinos have no spin-flip, and

$$C = \frac{1}{1 - \alpha \beta e^{2iqR/\hbar}},$$

$$A = \alpha C (1 - e^{2iqR/\hbar}),$$

$$E = -\alpha C e^{2iqR/\hbar},$$

$$G = (1 - \alpha \beta) C e^{i(q-p)R/\hbar}.$$
(28)

Note that the result (27) is a consequence of the fact that a massless particle has definite chirality even in the presence of torsion, in other words, the Hamiltonian $H_0 + V_T$ commutes with γ_5 . Therefore the probability of transmission of neutrino beam is (using (25) and (26))

$$|G|^2 = \frac{1}{1 + \frac{1}{2} \left(\frac{K}{cq}\right)^2 (1 - \sin 2qR/\hbar)}$$

$$= 1 - \frac{1}{2} \left(\frac{K}{E} \right)^2 (1 - \sin 2qR/\hbar) + \cdots, \tag{29}$$

where in the last step we assume that $E = pc \gg K$. In this way we find that the specific fraction of the incident neutrinos reflect due to this quantum–gravitational effect, and we have a flux reduction when a massless neutrino beam crosses through a region with non–zero spin polarization.

Now let us discuss the order of magnitude of this effect in different theoretical frameworks. 1. In ECD theory, in which we have worked until now, K is defined in eq.(21) and is equal to

$$K_{\text{ECD}}(\text{ev}) = 10^{-69} \rho(\text{cm}^{-3}).$$
 (30)

So for neutrinos with energy $E_{\nu} \sim O(1) \text{ev}$, this effect becomes significance only when $\rho \sim 10^{69} \text{ cm}^{-3}$ (see eq. (29)).

2. The torsionic contact interaction Lagrangian between two spin half particles is formally identical to the weak interaction Lagrangian and may be written in the (V-A) form, if at least one of the two fermions is massless. This suggest that the spin torsion coupling constant G_T , be also identified with the weak interaction Fermi constant, i.e., $G \to G_T \approx 10^{31} G$ [7,10-12]. Therefore

$$K_{V-A}(ev) = 10^{-38} \rho (cm^{-3}).$$
 (31)

So in this context, this effect is observable for matters with $\rho \sim 10^{38} \text{ cm}^{-3}$.

3. Inside the collapsed matter [13] and in the early stage of the universe [14], the gravity is in the strong regime, in which $G \to G_{SG} \approx 10^{39} G$ [15]. In these cases

$$K_{\rm SG}(\text{ev}) = 10^{-30} \rho(\text{cm}^{-3}).$$
 (32)

and the torsion–neutrino interaction becomes important for $\rho \sim 10^{30}~{\rm cm}^{-3}$.

4. Another approach in studying the interaction of torsion and fermions, is the effective field theory (EFT) approach which have been discussed in [16]. In this approach, the simplest action which includes all possible terms satisfying the symmetries of the theory has been considered. However, as far as one is interested in the low energy effect, the high derivative vertices are suppressed by the huge massive parameter which should be introduced for this purpose. In the heavy torsion limit, it can be shown that the Lagrangian of this model has the following contact four–fermion interaction term

$$\mathcal{L}_{\text{int}} = -\frac{\eta^2 \hbar^3}{2M_{\text{ts}}^2 c} (\bar{\psi} \gamma_5 \gamma^{\mu} \psi) (\bar{\psi} \gamma_5 \gamma_{\mu} \psi), \tag{33}$$

in which the parameter $\eta/(M_{\rm ts}c^2)$ has the following limit [16]

$$\frac{\eta}{M_{\rm ts}c^2} \le 1 \quad \text{Tev}^{-1} \tag{34}$$

Now if we substitute the expression (14) for A^{μ} in eq.(13), it is clear that we can recover the heavy torsion limit of EFT approach, if we replace $12\pi\hbar G/c^3$ by $(2\eta\hbar/(M_{\rm ts}c))^2$. In this way, the coupling constant of our model becomes

$$K_{\rm EFT} = 2\left(\frac{\eta}{M_{\rm ts}c^2}\right)^2 (\hbar c)^3 \rho. \tag{35}$$

Note that this replacement is in fact the replacement of the Planck scale energy (10¹⁶ Tev) with 1 Tev. Using $M_{\rm ts}c^2/\eta \simeq 1{\rm Tev}$, it is found that

$$K_{\rm EFT}(\rm ev) = 10^{-38} \rho (\rm cm^{-3}),$$
 (36)

and therefore in the context of EFT, the reflection of the neutrino beam is observable if $\rho \sim 10^{38} \ {\rm cm}^{-3}$.

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